Térence Bayen§, Yacine Chitour†, Frédédic Jean‡

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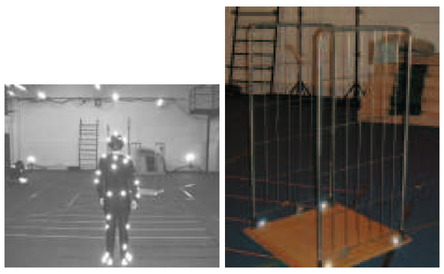
Modeling the human locomotion : a macroscopic approach

1 Introduction

The aim of this study is to understand the **human walking** and the **geometric shape** of locomotion trajectories. A person walking in an empty room from an initial point to a final point has many possible trajectories to perform this task. Among these trajectories, one is chosen. Our objective is to predict such a trajectory for potential applications in robotics. We present in this paper a model to describe the goal-oriented locomotion trajectories. The approach that has been chosen is macroscopic and takes advantage of an optimization principle, the Pontryagin Maximum Principle (PMP), which provides first-order necessary conditions.

2 Presentation of the problem

The study is based on experimental results made by Laumond and his colleagues (see [1],[2]). The trajectories followed by subjects walking from a point $I \in \mathbb{R}^2$ to a point $F \in \mathbb{R}^2$ have been recorded. Subjects start at an initial porch directed by $\theta_0 \in [0, 2\pi]$ and arrive at a final porch directed by $\theta_1 \in [0, 2\pi]$ (see pictures by Laumond). The two schemes below give a $2D$ view of the experiment and three possible strategies to go from I to F with $\theta(0) = \frac{\pi}{2}$, and $\theta(T) = \theta$.



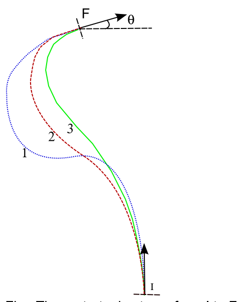


Fig : Three strategies to go from I to F.

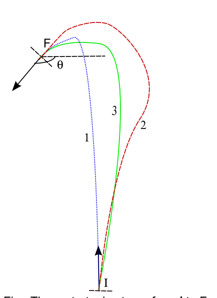


Fig : Three strategies to go from I to F.

3 Derivation of a model

3.1 Consequences of the experiments

The experiments of Laumond (see [2]) have lead to the following observations. Let (x, y) denote the coordinates of the trunk, and θ the direction of the speed vector.

- The **trunk** can be viewed as a **steering wheel** satisfying a nonholonomic constraint :

$$\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = 0, \qquad (1)$$

- The **curvature** κ of a locomotion trajectory is continuous.

From this observations, the following model can be derived to describe locomotion trajectories

$$\begin{cases} \dot{x} = \cos(\theta)u_1, \\ \dot{y} = \sin(\theta)u_1, \\ \dot{\theta} = \kappa u_1, \\ \dot{\kappa} = u_2. \end{cases} \qquad (2)$$

The subject is viewed as a system given by (2) **controlled** by its **linear speed** $u_1 \in [a, b]$, where $0 < a < b$, and by **the derivative of its curvature**, $u_2 \in [-c, c]$, where $c > 0$. The control $u := (u_1, u_2)$ belongs to a compact set $[a, b] \times [-c, c]$

3.2 The Pontryagin Maximum Principle (PMP)

To steer the trajectory (x, y, θ, κ) from $X_0 := (0, 0, \frac{\pi}{2}, 0)$ to $X_1 := (x_1, y_1, \theta_1, 0)$, we assume that the human brain **minimizes** a certain **criterium** along the trajectory. Such an **optimization principle** is common for studying neurogeometrical problems (see e.g. [3] concerning the arm movement and [4] for the problem of neuronal implementation). Several costs have been studied in the literature (jerk, minimum time...). It seems relevant to consider here the following cost :

$$C_u(T) := \frac{1}{2}\int_0^T u_1^2 + u_2^2, \qquad (3)$$

representing the **energy** of the subject during the interval of time $[0, T]$. The initial problem becomes an **optimal control problem** (OCP) : minimizing (3) among trajectories solution of (2) connecting X_0 to X_1. Notice that T is **not fixed** in the model.

Remark 1 *For any optimal trajectory of (OCP), a computation shows that $u_1 \equiv a$ on $[0, T]$, that is, a subject takes its* **lowest speed** *to go from X_0 to X_1, in order to minimize its energy.*

Remark 2 *System (2) is a* **non-holonomic** *system, the constraints of the form $f(\dot{x}, \dot{y}, \theta) = 0$ are* **non-integrable**. *The computation of optimal trajectories of (OCP) requires a precise analysis.*

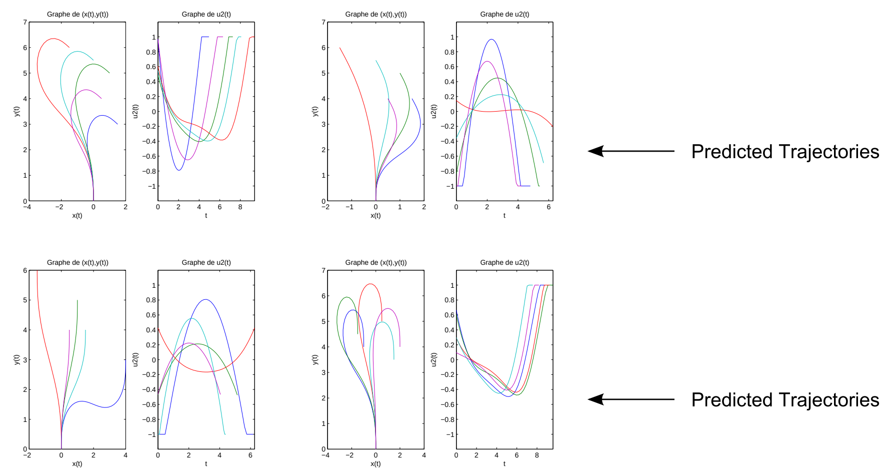
4 Simulation of locomotion trajectories

4.1 Properties of optimal trajectories

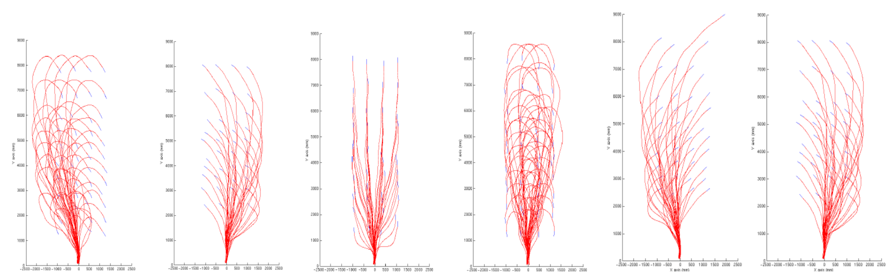
Theorem 1 *Let γ be a locomotion trajectory solution of (OCP) and $p(\cdot) \in \mathbb{R}^4$ a covector. Then, γ can be of type (i) or (ii) :*
(i). **Abnormal trajectory** *for which $u_2 \equiv \pm c$ on $[0, T]$ (γ is "Bang-Bang", that is a finite concatenation of clothoïds $\mathcal{B}_\pm$). The set of accessible points is of zero measure in $\mathbb{R}^2 \times \mathcal{S}^1 \times \mathbb{R}$.*
(ii). **Normal trajectory** *for which u_2 is regular on $[0, T]$: $u_2 = p_4 1_{-c \le p_4 \le c} \pm c 1_{|p_4| \ge c}$ (finite concatenation of clothoïds $\mathcal{B}_\pm$ and regular arcs $\mathcal{R}$). Moreover, there exists a compact $K \subset \mathbb{R}^2 \times \mathcal{S}^1 \times \mathbb{R}$ such that if $X_1 \notin K$, then only a normal trajectory connects X_0 and X_1.*

4.2 Numerical results

The graphs below have been obtained with `Matlab` by an **indirect shooting method** based on results of Theorem 1. Following the experiments of Laumond (see [1]), we have represented below numerical locomotion trajectories for $\theta_1 = -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{2}, -\frac{\pi}{2}$. For each direction, five targets have been chosen, and for each trajectory we have ploted the graph of u_2.



The figures below are real locomotion trajectories recorded by Laumond with same final direction $\theta_1 = -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{\pi}{2}, -\frac{\pi}{2}$ (see [1]).



It is to be remarked the similarity between the predicted and real locomotion trajectories. Moreover, most of the predicted trajectories obtained above are of type $\mathcal{B}_\pm\mathcal{R}\mathcal{B}_\pm$. This suggests that optimal trajectories are of this type.

5 Conclusion and perspectives

The model presented above shows two main characteristics of locomotion trajectories. Firstly, their structure is **simple**, that is, there is a **low number of switches** between an arc of clothoïd $\mathcal{B}_\pm$ and a regular arc $\mathcal{R}$. Secondly, the model above gives an accurate approximation of real trajectories, as the simulations are close to the shape of real trajectories. The study of (2) offers numerical and theoretical perspectives to confirm these observations and to improve the optimal synthesis of (OCP).

6 References

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§ *Université Montpellier 2*, Département de Mathématiques, Place Eugène Bataillon 34095 Montpellier, France (`tbayen@math.univ-montp2.fr`).

† *Université Paris XI*, Laboratoire des signaux et systèmes (L2S) Supélec, 3 rue Joliot-Curie, 91190 Gif-sur-Yvette, France (`chitour@lss.supelec.fr`).

‡ *ENSTA, UMA*, 32 boulevard Victor, 75739 Paris, France (`Frederic.jean@ensta.fr`).