digiteo
Research in science and technology of information
www.digiteo.fr

W. K. Park, H. Ammari and D. Lesselier

Poster n°2008-08

Non-Iterative Imaging of Electromagnetic Thin Inclusions

ABSTRACT

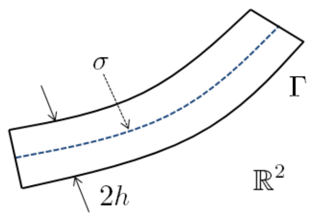
We consider a non-iterative MUSIC-type imaging algorithm for reconstructing thin, curve-like penetrable inclusions in a two-dimensional homogeneous space. It is based on an appropriate asymptotic formula of the scattering amplitude. Operating at fixed non-zero frequency, it yields the shape of the inclusion from scattered fields. Numerical implementation shows that it is fast and efficient.

1 Direct Scattering Problem

1.1 Configuration

- Penetrable thin inclusion localized in the neighborhood of a smooth curve σ:

$$\Gamma = \{x + \eta n(x) : x \in \sigma,\ \eta \in (-h, h)\}.$$



- Piecewise constant dielectric permittivity $\varepsilon(x)$ and wavenumber $k(x)$ defined as

$$\varepsilon(x) = \begin{cases} \varepsilon_0 & \text{for} \quad x \in \mathbb{R}^2\backslash\overline{\Gamma} \\ \varepsilon & \text{for} \quad x \in \Gamma \end{cases} \quad \text{and} \quad k(x) = \begin{cases} k_0 = \omega\sqrt{\varepsilon_0} & \text{for} \quad x \in \mathbb{R}^2\backslash\overline{\Gamma} \\ k = \omega\sqrt{\varepsilon} & \text{for} \quad x \in \Gamma \end{cases}.$$

- $\{\hat{y}_j\}_{j=1}^N, \{\theta_l\}_{l=1}^N \subset S^1$: set of observation and incident directions, respectively.

1.2 Asymptotic expansion of the scattering amplitude

- $u(x)$: time-harmonic total electric field satisfying the Helmholtz equation

$$\triangle u(x) + k(x)^2 u(x) = 0 \quad \text{for} \quad x \in \mathbb{R}^2$$

with standard continuity conditions at inclusion boundaries and at infinity.

- The scattering amplitude $K(\hat{y}, \theta)$ satisfies

$$u(y) - u_0(y) = u_s(y) = \frac{e^{ik_0|y|}}{\sqrt{|y|}} K(\hat{y}, \theta) + o\left(\frac{1}{\sqrt{|y|}}\right)$$

as $|y| \to \infty$ uniformly on $\hat{y} = \frac{y}{|y|}$.

- The asymptotic formula for the scattering amplitude

$$K(\hat{y}, \theta) = h\frac{k_0^2(1+i)}{4\sqrt{k_0\pi}} \int_\sigma (\varepsilon - \varepsilon_0) e^{ik_0(\theta - \hat{y})\cdot x} d\sigma(x) + o(h).$$

2 MUSIC-Type Imaging Algorithm

- $K = (K(-\theta_j, \theta_l))_{j,l=1}^N \in \mathbb{C}^{N\times N}$: Multi-Static Response (MSR) matrix
- Singular value decomposition of K : $K = VSW^T$
- Singular values: $\underbrace{\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M}_{\text{non-zero singular values}} \geq \underbrace{\lambda_{M+1} \geq \cdots \geq \lambda_N}_{\text{zero or very small}}$
- Singular vectors: $V = [\underbrace{v_1, v_2, \cdots v_M}_{\text{signal}}, \underbrace{v_{M+1} \cdots, v_N}_{\text{noise}}]$
- If $z \in \sigma$, $g(z) = \left(e^{ik_0\theta_1\cdot z}, e^{ik_0\theta_2\cdot z}, \cdots, e^{ik_0\theta_N\cdot z}\right)^T$ belongs to the signal sub-space of K
- Orthogonal projection onto the noise subspace: $P_{\text{noise}}(f) = \sum_{j>M} v_j \overline{v}_j^T f.$
- An image of points $z \in \sigma$ by plotting the value

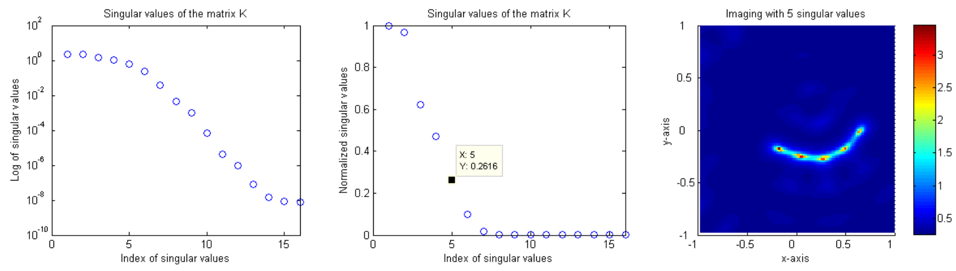
$$W(z) = \frac{1}{||P_{\text{noise}}(g(z))||}$$

the resulting plot is expected to exhibit large peaks at points on Γ.

3 Numerical Examples

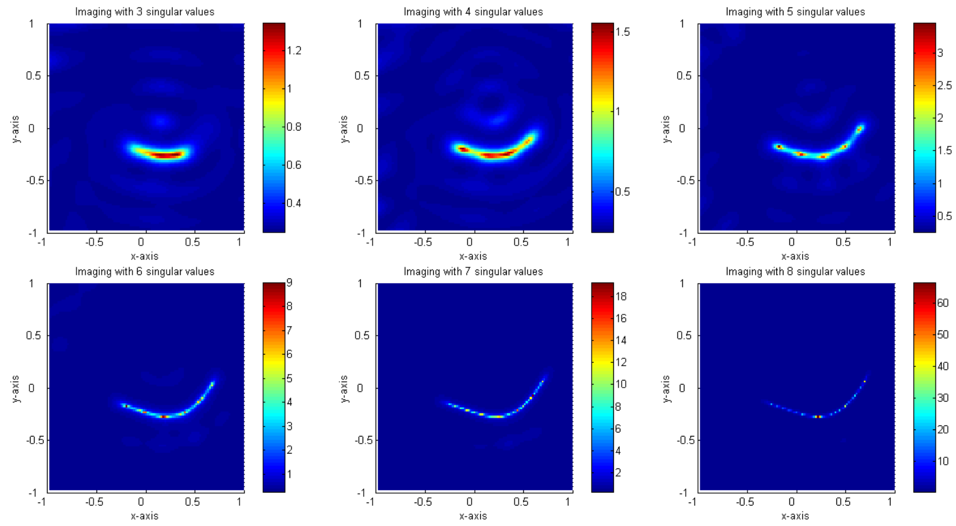
3.1 Normalizing the singular values

- All singular values are normalized with respect to the one of maximum amplitude.
- One can easily choose non-zero singular values to discriminate the noise subspace from the signal subspace.



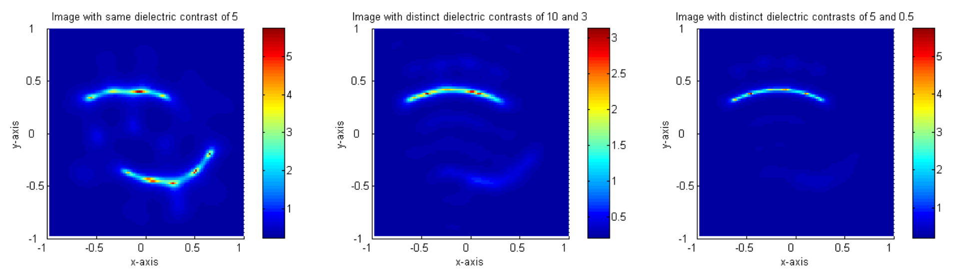
3.2 Influence of the number of singular values

- Too small a number of singular values leads to a blurred image, too high a number of singular values to a perturbed one.



3.3 Extension to multiple inclusions

- The algorithm can be applied to the imaging of two (or more) inclusions.
- In contrast with the single inclusion case, if an inclusion has a much smaller value of permittivity than the other, it does not significantly affect the MSR matrix and the inclusion is not retrieved via the MUSIC-type algorithm.



Concluding Remarks

- MUSIC-type algorithm is fast and effective for imaging of dielectric thin inclusions.
- This algorithm successfully applies to a purely magnetic one and to combination cases.
- These non-iterative imaging results obtained at low computational cost could be an initial guess of a level-set evolution or a Newton-type iteration algorithm.
- This algorithm works as well for impenetrable screens, yet mathematical formulation requires further investigation.
- Extension to 3-D screens is expected.

Contacts : won-kwang.park@lss.supelec.fr · habib.ammari@polytechnique.fr · dominique.lesselier@lss.supelec.fr

L2S · CMAP · îledeFrance · CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE · Supélec · ÉCOLE POLYTECHNIQUE · UNIVERSITÉ PARIS-SUD 11